the local velocity of the mass and that the limits of integration are functions of time

At approximately the same time (although unknown to author), Thorpe[2] established a similar theorem for piecewise continuous masses Thorpe's theorem, although not as general as Bottaccini's, was particularly interesting since he found a practical equation, whereas Bottaccini only proved a principle

Thorpe showed that, for an arbitrary piecewise continuous mass moving in an arbitrary manner under external forces, the forces acting on the portion of the mass within an arbitrarily selected control volume are given by

$$\frac{d}{dt}\int_V \rho \mathbf{U} d\tau = \Sigma \mathbf{F} - \int_S \rho \mathbf{U}(\mathbf{U} - \mathbf{Y})\, d\mathbf{S} \qquad (3)$$

in which ρ is the local mass density, V is the arbitrary volume, and $\mathbf{Y}$ is the local velocity of the boundary surface S This relative motion expression can be brought into agreement with Eq (2) by using the definition of momentum given in Eq (1), as was recognized by Thorpe[3] Since in Eq (1) the integral is to be taken over the mass, then the velocity of the bounding surface must be the velocity of the mass on the boundary With this definition, $\mathbf{U} = \mathbf{Y}$, and Eq (3) becomes

$$\frac{d}{dt}\int_V \rho \mathbf{U} d\tau = \Sigma \mathbf{F}$$

For piecewise continuous masses, Eq (1) becomes

$$\mathbf{G} = \int_V \mathbf{U}\frac{dm}{d\tau}\, d\tau = \int_V \rho \mathbf{U} d\tau$$

which shows that Eqs (2) and (3) are identical Equation (3), however, is admirably suited for computations on piecewise continuous masses For highly discontinuous masses and for masses defined on sets of measure zero, the reader is referred to Ref 1

References

[1] Bottaccini, M "An alternate interpretation of Newton's second law," AIAA J **1**, 927–928 (1963)

[2] Thorpe, J F, "On the momentum theorem for a continuous system of variable mass" Am J Phys **30**, 637–640 (1962)

[3] Thorpe, J F letter to author (July 1963)

Effect of Radiation on Ammonium Perchlorate Propellants

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THIS laboratory has recently completed a series of experiments to determine the effects of radiation on propellants The propellant strands were obtained from the vendors cited and irradiated using a 2-Mev Van de Graaff electron accelerator After exposure to the doses indicated in Table 1, burning rates and tensile measurements were made

It is seen that, in many cases, drastic changes in burning rates and tensile strengths occurred upon radiolysis The mechanisms by which these changes are brought about are being studied in a continuing program in which the individual components of the propellant recipe are being irradiated and incorporated into nonirradiated mixes

Table 1 Effect of radiation on ammonium perchlorate propellants

Propellant	Radiation dose, mrad	Burning rate,[a] in /sec	Tensile strength,[b] psi
Polysulfide, Thiokol TP-L-3014	0	0 0593 ± 0 0006	249 ± 11
	10	0 0593 ± 0 0060	156 ± 9
	50	0 0549 ± 0 0025	51 ± 13
Polysulfide, Thiokol TP-L-3014a	0	0 0582 ± 0 0003	136 ± 4
	20	0 0548 ± 0 0005	138 ± 15
	50	0 0568 ± 0 0006	62 ± 6
Hydrocarbon, Thiokol TP-H-3062	0	0 0422 ± 0 0003	91 ± 4
	20	0 0428 ± 0 0004	168 ± 7
	50	0 0425 ± 0 0004	145 ± 7
Polyurethane, Thiokol TP-6 3129	0	0 0347 ± 0 0002	54 ± 3
	20	0 0355 ± 0 0002	56 ± 3
	50	0 0371 ± 0 0004	40 ± 2
Polyacrylonitrile Hercules HES 6648	0	0 0660 ± 0 0025	190 ± 8
	10	0 0700 ± 0 0024	72 ± 2
	50	0 0860 ± 0 0027	56 ± 2
Polyethyl acrylate, Hercules HES 6420	0	0 0412 ± 0 0004	111 ± 10
	10	0 0447 ± 0 0005	67 ± 6
	50	0 0486 ± 0 0010	30 ± 4
Cellulose acetate, Hercules HES 5808	0	0 0325 ± 0 0010	541 ± 75
	10	0 0323 ± 0 0006	341 ± 34

[a] Number of determinations = 10–20
[b] Number of determinations = 5

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Shell Buckling and Nonconservative Forces

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IN a recent note Niedenfuhr[1] advanced the suggestion that the wide scatter of observed buckling loads of pressurized shells might be attributed to the presence of nonconservative generalized forces The suggestion was based on the statement that a mechanical system that is acted upon by nonconservative generalized forces may buckle dynamically as well as statically This statement, in turn, was supported by the example of a two-degree-of-freedom system subjected to one conservative and one nonconservative load

It appears to these writers that the statement just quoted, which has limited validity, is not applicable in the sense envisaged by Niedenfuhr, as will be indicated below For a given ratio of the two loads introduced, it is of course possible to calculate a static and a dynamic load, but the physically meaningful one, in general, will be only the lower one If it is the static one, the system will be displaced into a position of static equilibrium corresponding to the actual value of the (supercritical) load If, on the other hand, the stability is lost dynamically, under a load larger than the critical one, the system will vibrate with a definite frequency and with an exponentially increasing amplitude until failure is reached Thus, in the case considered by Niedenfuhr there is no possibility of natural experimental scatter for fixed loading ratios

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